Kaushik Bhattacharya[1],Avijit K. Ganguly[1],Sushan Konar[2]
[1] *Saha Institute of Nuclear Physics, 1/AF, Bidhan-Nagar, Calcutta 700064, India*
[2] *IUCAA, Post Bag 4, Ganeshkhind, Pune 411007, India*
e-mail : kaushikb@theory.saha.ernet.in, avijit@theory.saha.ernet.in, sushan@iucaa.ernet.in

Neutrino-photon processes, forbidden in vacuum, can take place in presence of a thermal medium or an external electro-magnetic field, mediated by the corresponding charged leptons (real or virtual). The effect of a medium or an electromagnetic field is two-fold - to induce an effective $\nu-\gamma$ vertex and to modify the dispersion relations of all the particles involved to render the processes kinematically viable. It has already been noted that in presence of a thermal medium such an electromagnetic interaction translates into the neutrino acquiring a small effective charge. In this work, we extend this concept to the case of a thermal medium in presence of an external magnetic field and calculate the effective charge of the neutrino to odd orders in the magnetic field. We find that the effective charge is direction dependent which is a direct effect of magnetic field breaking the isotropy of the space.

I. INTRODUCTION

Neutrino mediated processes are of great importance in astrophysics and cosmology [1]. Among them, processes like $\nu \to \nu\gamma$, and the cross-related reactions, $\gamma\gamma \to \nu\bar{\nu}$, $\nu\bar{\nu} \to e^+e^-$ etc. become extremely important in medium —in particular reactions those are highly suppressed/forbidden in vacuum, becomes allowed in medium, acquiring great importance in astrophysical situations involving the physics of compact steller objects e.g white-dwarfs, Red-Giants, or neutron-stars.

On the other hand all these compact objects are known to be associated with a magnetic field whose strength varies from object to object. In particular, all the neutron stars or pulsars are known to be associated with a magnetic field whose strength seems to be dependent on the age of the star. The survey of pulsars in the galactic plane show that for pulsars of the age of 10^9 or 10^{10} years– are usually associated with surface field strengths of the order of 10^8 Gauss. On the other hand the surface magnetic field strength for relatively younger ones (age $10^5 - 10^6$ years) are of the order of 10^{12}Gauss [2]. It is usually conjectured –taking into account the conservation of surface magnetic field of a proto-neutron star, that during a supernova collapse, the magnetic field strength in some regions inside the nascent star, can reach upto $m_e^2 \sim e\mathcal{B}$ or more. Where m_e is the mass of an electron. Hence forth we would refer to field strengths of this magnitude as critical field strength $\mathcal{B}_c$.

It is also worth mentioning at this stage that, in recent years there are quite a few suggestions in the literature, that there might be stars whose field strength can exceed that of $\mathcal{B}_c$. These class of compact objects come under the name of Soft Gamma Repeaters (SGR). In references [3–5] it has been claimed—based on the spin down time scale– that, these are newly born neutron stars, whose surface magnetic field , (e.g SGR 0526-66 SGR 1806-20) is claimed to be as high as 10^{15} Gauss. These are termed as magnetars [6]. The anamolous Xray pulsars are also included in this family of stars [22].

In view of this , studies of neutrino properties in a magnetized medium seems to be interesting for astrophysical situations. It is widely believed that neutrinos play a crucial role during steller evolution. Because of effective neutrino photon interaction in a medium, it is possible that the neutrinos might dump a fraction of their energy inside the star during steller evolution. For instance in a type II supernovae collapse [20] neutrinos produced deep inside the protoneutron star surge out carrying an effective energy $\sim 10^{52}$ ergs/sec. It is conjectured that the neutrinos deposit some fraction of its energy during the explosion through different kinds of neutrino electromagnetic interactions, e.g, $\nu \to \nu\gamma$, $\nu\bar{\nu} \to e^+e^-$ –to name a few (It is important to note here, that all these processes are of order G_F^2). However the amount of energy dumped by these mechanisms to the mantle of the proto-neutron star, seem to be barely sufficient to blow the outer part of the same.

Recently in a set of papers [23] it was conjectured that, the freely streaming neutrinos from the supernova core interact with the nonrelativistic electrons present in the outer part of the core through collective interactions– what is known in the standard plasma physics parlance as two stream instability.

Neutrino wind driven [23] instability is believed to be responsible for blowing up of the mantle of the supernova core. Although according to standard model of particle physics neutrinos do not carry any charge, but the curious fact is neutrinos in a medium acquires some induced charge because of the following reason. When a neutrino moves inside a thermal medium composed of electrons and positrons, they interact with these background particles. The background electrons and positrons themselves have interaction with the electromagnetic fields, and this fact gives rise to an effective coupling of the neutrinos to the photons. In this circumstances the neutrinos may acquire an "effective electric charge" through which they interact with the ambient electrically charged plasma.

The main purpose of this paper is to provide an expression for the effective neutrino charge in a magnetized plasma to odd orders in the external magnetic field, in a later publication we have the plan of calculating the effective charge of the neutrinos to even powers of the external field. In the conclusion we would briefly discuss how the induced neutrino charge helps to develop the neutrino wind driven instability (i.e the two stream instability) responsible for deposition of energy in the supernova mantle as is put forward in [23], [25], [24]. Although we would like to point out that this particular mechanism is frought with controversy; with different groups claiming to have obtained different results. Since the purpose of this paper is to deal with neutrino charge in a magnetized medium, so we would not go into the details of this issue.

The effective charge of the neutrino has been calculated previously by many authors [10,18,19,13]. In this note extend their analysis to the situation where there is a magnetized medium. Firstly the results of our analysis shows that the neutrino effective charge in a magnetized medium ($e^\nu_{\rm eff}(e\mathcal{B})$) is proportional to $\mathcal{B}/\mathcal{B}_c$, (where $\mathcal{B}_c \sim 10^{13}$ Gauss) which decides whether $e^\nu_{\rm eff}(e\mathcal{B})$ can exceed that of $e^\nu_{\rm eff}(e\mathcal{B}=0)$. The second interesting part is,the effective charge can be direction dependent, which is basically a manifestation of the fact that an external field always breaks the isotropy of the medium.

In order to find it out we first obtain the effective neutrino photon Lagrangian (details would be communicated in a separate publiction [17]) in a magnetized medium by integrating out the intermediate charged fermion lines, and from there get the expression for the the corresponding neutrino photon vertex in the static long wave length limit, i.e for the photon $k_0 = 0$ and $|\vec{k}| \to 0$.

II. FORMALISM

The off-shell electromagnetic vertex function Γ_λ is defined in such a way that, for on-shell neutrinos, the $\nu\nu\gamma$ amplitude is given by:

$$\mathcal{M} = -i\bar{u}(q')\Gamma_\lambda u(q) A^\lambda(k), \tag{1}$$

where, k is the photon momentum. Here, $u(q)$ is the the neutrino spinor and A^μ stands for the electromagnetic vector potential. In general, Γ_λ would depend on k, and the characteristics of the medium. We shall, in this work, consider neutrino momenta that are small compared to the masses of the W and Z bosons. We can, therefore, neglect the momentum dependence in the W and Z propagators, which is justified if we are performing a calculation to the leading order in the Fermi constant, G_F. In this limit four-fermion interaction is given by the following effective Lagrangian:

$$\mathcal{L}_{\rm eff} = \frac{1}{\sqrt{2}} G_F \overline{\nu}\gamma^\mu(1+\gamma_5)\nu \bar{l}_\nu \gamma_\mu (g_{\rm V} + g_{\rm A}\gamma_5) l_\nu \,, \tag{2}$$

where, ν and l_ν are the neutrino and the corresponding lepton field respectively. For electron neutrinos,

$$g_{\rm V} = 1 - (1 - 4\sin^2\theta_{\rm W})/2, \tag{3}$$

$$g_{\rm A} = 1 - 1/2; \tag{4}$$

where the first terms in $g_{\rm V}$ and $g_{\rm A}$ are the contributions from the W exchange diagram and the second one from the Z exchange diagram. Then the amplitude effectively reduces to that of a purely photonic case with one of the photons replaced by the neutrino current, as seen in the diagram in fig. 1. Therefore, Γ_ν is given by:

$$\Gamma_\nu = -\frac{1}{\sqrt{2}e} G_F \gamma^\mu (1+\gamma_5)\left(g_{\rm V}\Pi_{\mu\nu} + g_{\rm A}\Pi^5_{\mu\nu}\right), \tag{5}$$

where, $\Pi^5_{\mu\nu}$ represents the vector-axial vector coupling and $\Pi_{\mu\nu}$ is the polarisation tensor. In an earlier paper [26] (paper-I henceforth) we have analysed the structure of $\Pi^{\mu\nu}$, in a background medium in presence of an uniform external magnetic field to all odd orders (in $\mathcal{B}$), calculated at the 1-loop level. We shall use the results of paper-I here to obtain the total effective charge of the neutrinos under equivalent conditions. Because of the electromagnetic current conservation, for the polarisation tensor, we have the following gauge invariance condition:

$$k^\mu \Pi_{\mu\nu} = 0 = \Pi_{\mu\nu} k^\nu. \tag{6}$$

Same is true for the photon vertex of fig. 1 and we have

$$\Pi^5_{\mu\nu} k^\nu = 0\,. \tag{7}$$

The effective charge of the neutrinos is defined in terms of the vertex function by the following relation [18]:

$$e^\nu_{\rm eff} = \frac{1}{2q_0}\,\bar{u}(q)\,\Gamma_0(k_0 = 0, \mathbf{k} \to 0)\,u(q)\,. \tag{8}$$

For massless Weyl spinors this definition can be rendered into the form:

$$e^\nu_{\rm eff} = \frac{1}{2q_0}\,{\rm tr}\left[\Gamma_0(k_0 = 0, \mathbf{k}\to 0)\,(1+\lambda\gamma^5)\,\not{q}\right] \tag{9}$$

where $\lambda = \pm 1$ is the helicity of the spinors.

It can be seen from eq.(9) that, in general, the effective neutrino charge depends on $\Pi_{\mu\nu}(q)$ as well as on $\Pi^5_{\mu\nu}(q)$. But in [18] it has been shown that the non-zero contribution to the effective charge, in presence of a thermal medium, comes only from the $\Pi_{\mu\nu}(k)$ part. On the other hand, in the case of an external magnetic field in vacuum, it has can be shown [26] that $\Pi_{\mu\nu}(k)$ vanishes to odd orders in $\mathcal{B}$. Hence, to odd orders in the external magnetic field the contribution to the effective charge comes only from $\Pi^5_{\mu\nu}(k)$ in this case.

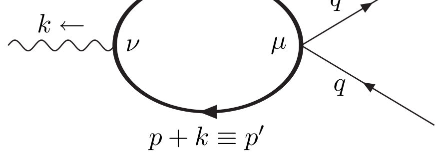

FIG. 1. One-loop diagram for the effective electromagnetic vertex of the neutrino in the limit of infinitely heavy W and Z masses.

Similarly, $\Pi_{\mu\nu}(k)$ to odd orders in $\mathcal{B}$ is proportional to $\varepsilon_{\mu\nu\alpha\beta}k^\beta$ in a magnetised medium (see eq.(5.21) of paper-I) and goes to zero in the limit $k_0 = 0, \mathbf{k} \to 0$. Therefore, in a magnetised medium, the non-zero contribution to the effective charge of the neutrinos come solely from $\Pi^5_{\mu\nu}(k)$, to odd orders in the external field, and the effective charge of the neutrinos is given by:

$$e^\nu_{\text{eff}} = -\frac{1}{2q_0}\frac{G_F}{\sqrt{2}e}\, g_{\rm A}\Pi^5_{\mu 0}(k_0 = 0, \mathbf{k} \to 0) \times \operatorname{tr}\left\{\gamma^\mu(1+\gamma_5)\left(1+\lambda\gamma^5\right)\not{k}\right\} . \tag{10}$$

III. RESULTS FROM THE 1-LOOP DIAGRAM

Since we investigate the case of a pure magnetic field, it can be taken in the z-direction without any further loss of generality. We denote the magnitude of this field by $\mathcal{B}$. Ignoring at first the presence of the medium, the electron propagator in such a field can be written down following Schwinger's approach [27–29]:

$$iS^V_B(p) = \int_0^\infty ds\, e^{\Phi(p,s)}\, G(p,s)\,, \tag{11}$$

where Φ and G are as given below

$$\Phi(p,s) \equiv is\left(p^2_\parallel - \frac{\tan(e\mathcal{B}s)}{e\mathcal{B}s}\,p^2_\perp - m^2\right) - \epsilon|s|\,, \tag{12}$$

$$\begin{aligned} G(p,s) &\equiv \frac{e^{ie\mathcal{B}s\sigma_z}}{\cos(e\mathcal{B}s)}\left(\not{p}_\parallel + \frac{e^{-ie\mathcal{B}s\sigma_z}}{\cos(e\mathcal{B}s)}\not{p}_\perp + m\right) \\ &= \left[(1 + i\sigma_z \tan e\mathcal{B}s)(\not{p}_\parallel + m) + (\sec^2 e\mathcal{B}s)\not{p}_\perp\right], \end{aligned} \tag{13}$$

where

$$\sigma_z = i\gamma_1\gamma_2 = -\gamma_0\gamma_3\gamma_5\,, \tag{14}$$

and we have used,

$$e^{ie\mathcal{B}s\sigma_z} = \cos\, e\mathcal{B}s + i\sigma_z \sin\, e\mathcal{B}s\,. \tag{15}$$

Also here

$$\begin{aligned} \not{p}_\parallel &= \gamma_0 p^0 + \gamma_3 p^3 \\ \not{p}_\perp &= \gamma_1 p^1 + \gamma_2 p^2 \\ p^2_\parallel &= p^2_0 - p^2_3 \\ p^2_\perp &= p^2_1 + p^2_2. \end{aligned}$$

Of course in the range of integration indicated in eq. (11) s is never negative and hence $|s|$ equals s. In a background medium, the above propagator is modified to:

$$iS(p) = iS^V_B(p) + S^\eta_B(p)\,, \tag{16}$$

where

$$S^\eta_B(p) \equiv -\eta_F(p)\left[iS^V_B(p) - i\overline{S}^V_B(p)\right]\,, \tag{17}$$

and

$$\overline{S}^V_B(p) \equiv \gamma_0 S^{V\dagger}_B(p)\gamma_0\,, \tag{18}$$

for a fermion propagator, such that

$$S^\eta_B(p) = -\eta_F(p)\int_{-\infty}^\infty ds\; e^{\Phi(p,s)}G(p,s)\,. \tag{19}$$

And $\eta_F(p)$ contains the distribution function for the fermions and the anti-fermions:

$$\begin{aligned} \eta_F(p) &= \Theta(p\cdot u) f_F(p,\mu,\beta) \\ &\quad + \Theta(-p\cdot u) f_F(-p,-\mu,\beta)\,. \end{aligned} \tag{20}$$

Here, f_F denotes the Fermi-Dirac distribution function:

$$f_F(p,\mu,\beta) = \frac{1}{e^{\beta(p\cdot u-\mu)}+1}\,, \tag{21}$$

and Θ is the step function given by:

$$\begin{aligned} \Theta(x) &= 1,\ \text{for}\ x > 0\,, \\ &= 0,\ \text{for}\ x < 0\,. \end{aligned}$$

The expression for $\Pi^5_{\mu\nu}$ as computed using the propagator given in eq.(16) is,

$$\begin{aligned} i\Pi^5_{\mu\nu}(k) &= -(-ie^2)(-1)\int\frac{d^4p}{(2\pi)^4}\int_{-\infty}^\infty ds\, e^{\Phi(p,s)} \\ &\quad \int_0^\infty ds' e^{\Phi(p',s')}\left[\operatorname{Tr}\left[\gamma_\mu\gamma_5 G(p,s)\gamma_\nu G(p',s')\right]\eta_F(p)\right. \\ &\quad \left. + \operatorname{Tr}\left[\gamma_\mu\gamma_5 G(-p',s')\gamma_\nu G(-p,s)\right]\eta_F(-p)\right] \\ &= -(-ie^2)(-1)\int\frac{d^4p}{(2\pi)^4}\int_{-\infty}^\infty ds\, e^{\Phi(p,s)} \\ &\quad \times\int_0^\infty ds'\, e^{\Phi(p',s')}\mathrm{R}_{\mu\nu}(p,p',s,s') \end{aligned} \tag{22}$$

where $\mathrm{R}_{\mu\nu}(p,p',s,s')$ contains the trace part.

A. $\mathbf{R}_{\mu\nu}$ to odd orders in magnetic field

We perform the calculations in the rest frame of the medium where $p\cdot u = p_0$. Thus the distribution function does not depend on the spatial components of p. In this case to odd powers in external field $\mathcal{B}$

$$\begin{aligned}\mathrm{R}^{(o)}_{\mu\nu} &= 4i\eta_+(p_0)\Bigg[-\varepsilon_{\mu\nu 12}\left\{\frac{\sec^2(e\mathcal{B}s)\tan^2(e\mathcal{B}s')}{\tan(e\mathcal{B}s)+\tan(e\mathcal{B}s')}k_\perp^2\right.\\
&+ \left.(k\cdot p)_\parallel(\tan(e\mathcal{B}s)+\tan(e\mathcal{B}s'))\right\}\\
&+ 2\varepsilon_{\mu 12\alpha_\parallel}(p'_{\nu_\parallel}p^{\alpha_\parallel}\tan(e\mathcal{B}s)+p_{\nu_\parallel}p'^{\alpha_\parallel}\tan(e\mathcal{B}s'))\\
&+ g_{\mu\alpha_\parallel}k_{\nu_\perp}\left\{p^{\widetilde{\alpha}_\parallel}(\tan(e\mathcal{B}s)-\tan(e\mathcal{B}s'))\right.\\
&- \left.k^{\widetilde{\alpha}_\parallel}\frac{\sec^2(e\mathcal{B}s)\tan^2(e\mathcal{B}s')}{\tan(e\mathcal{B}s)+\tan(e\mathcal{B}s')}\right\}\\
&+ \{g_{\mu\nu}(p\cdot\widetilde{k})_\parallel + g_{\nu\alpha_\parallel}p^{\widetilde{\alpha}_\parallel}k_{\mu_\perp}\}\\
&\times (\tan(e\mathcal{B}s)-\tan(e\mathcal{B}s'))\\
&+ g_{\nu\alpha_\parallel}k^{\widetilde{\alpha}_\parallel}p_{\mu_\perp}\sec^2(e\mathcal{B}s)\tan(e\mathcal{B}s')\Bigg]\end{aligned}\tag{23}$$

where, in our convention

$$a_\mu b^{\widetilde{\mu}_\parallel} = a_0 b^3 + a_3 b^0$$

and

$$(a\cdot\widetilde{b})_\parallel = a^0 b^3 - a^3 b^0$$

and $\eta_+(p_0)$ contains the information about the distribution functions, given by

$$\eta_+(p_0) = \eta_F(p_0) + \eta_F(-p_0) \tag{24}$$

which is an even function of p_0.

IV. GENERAL ANALYSIS OF THE TENSORIAL STRUCTURE OF $\Pi_{\mu\nu}$ AND $\Pi^5_{\mu\nu}$.

In vacuum

$$\Pi_{\mu\nu}(k) = [g_{\mu\nu}k^2 - k_\mu k_\nu]\,\Pi(k^2) \tag{25}$$

where $\Pi(k^2)$ is zero when $k_0 = 0, \bar{k} \to 0$. As a result, in the specified momentum limit $\Pi_{\mu\nu}(k)$ reduces to zero.

The other contribution to charge coming from $\Pi^5_{\mu\nu}(k)$ in vacuum vanish. This can be understood from the general form factor analysis. This two indexed tensor can be written in terms of $g_{\mu\nu}$ and $\epsilon_{\mu\nu\lambda\sigma}$, and k_λ. The parity structure of the theory forbids appearence of $g_{\mu\nu}$, so the only available tensor we have at hand is, $\epsilon_{\mu\nu\lambda\sigma}$. Now only the other vector we have at hand to make it a tensor of rank two is k_λ; so as we contract $\epsilon_{\mu\nu\lambda\sigma}$ with k_λ, k_σ, the corresponding term vanishes. And this completes the proof, that in vacuum effective charge of the neutrino is zero.

On the other hand, in a medium the polarisation tensor can be expanded in terms of form factors as [10]

$$\Pi_{\mu\nu}(k) = \Pi_T\,T_{\mu\nu} + \Pi_L\,L_{\mu\nu} + \Pi_P\,P_{\mu\nu} \tag{26}$$

where

$$T_{\mu\nu} = \widetilde{g}_{\mu\nu} - L_{\mu\nu} \tag{27}$$

$$L_{\mu\nu} = \frac{\widetilde{u}_\mu\widetilde{u}_\nu}{\widetilde{u}^2} \tag{28}$$

$$P_{\mu\nu} = \frac{i}{\mathcal{Q}}\varepsilon_{\mu\nu\alpha\beta}k^\alpha u^\beta \tag{29}$$

with

$$\widetilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \tag{30}$$

$$\widetilde{u}_\mu = \widetilde{g}_{\mu\rho}u^\rho \tag{31}$$

$$\mathcal{Q} = \sqrt{(k\cdot u)^2 - k^2} \tag{32}$$

in the rest frame of the medium $u^\mu = (1,0,0,0)$. It is easy to see that the longitudinal projector $L_{\mu\nu}$ is not zero in the limit $k_0 = 0, \bar{k} \to 0$ and Π_L is also not zero in the above mentioned limit. This fact is responsible for giving nonzero contribution to the effective charge of neutrino. The tensor structure of $\Pi^5_{\mu\nu}$ in a medium is of the form $\varepsilon_{\mu\nu\alpha\beta}\,k^\alpha u^\beta$ and this part although is not zero, dosent contribute to the zeroth component of Γ_ν.

In a pure magnetic field in absence of any medium the calculation of both the quantities on which Γ_ν depends has been done [9], and both $\Pi_{\mu\nu}$ and $\Pi^5_{\mu\nu}$ tends to zero in the external momentum going to zero limit giving no contribution to the neutrino effective charge in a magnetic field.

In order to elaborate on this, we would like to draw attention to the fact that –the magnetic field dependent part of the polarization tensor, with odd powers of magnetic field would vanish in absence of matter, as a consequence of Furrys theorem. The only contribution to such terms would come from matter + magnetic field dependent part (One can show again from charge conjugation invariance that $\Pi\mu\nu$ with odd powers of magntic field is nonzero only in presence of a medium) and this contribution is proportional to (see eqn. (5.21) Paper-I) $\epsilon_{\mu\nu\alpha_{||}\beta}k^\beta$, which vanishes in the zero frequency long wavelength limit.

A similar analysis of $\Pi^5_{\mu\nu}$, in medium without magnetic field would show [18], that its contribution to $e^\nu_{\rm eff} = 0$ leaving us with the possibility getting contributions from just magnetized vacuum part and magnetized plasma part.

The contributions from magnetized vacuum part for $\Pi^5_{\mu\nu}(k)$ was performed in, [9], with the result,

$$\begin{aligned}\Pi^5_{\mu\nu}(k) = \frac{i}{(4\pi)^2}\int_0^\infty ds\int_{-1}^1 dv/2e^{-is\chi}\Bigg[(1-v^2)k^\mu_{\parallel}e\tilde{F}k_\nu\\
+R\Big[-k^2_{\perp}e\tilde{F}^{\mu\nu} + k^\mu_{\perp}e\tilde{F}k_\nu + k^\nu_{\perp}e\tilde{F}k_\mu\Big]\Bigg]\end{aligned}\tag{33}$$

Where,

$$\chi = m^2 + \frac{(1-v^2)}{4}k_\parallel^2 + \frac{\cos vz - cos z}{2z\sin z}$$
$$R = \frac{1 - v\sin vz\sin z - \cos vz\cos z}{\sin^2 z} \quad (34)$$

Where, $z = eBs$ and also note that the metric used in eqn.[33] $g^{\mu\nu} = diag(-,+,+,+)$ and $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\lambda\sigma}F_{\lambda\sigma}$. It is obvious from the general tensor structure that in the zero frequency and long wave length the all of these tensors would vanish hence $\Pi^5_{\mu\nu}$–in a magnetic field would not contribute to neutrino effective charge.

Our calculation is in a medium with a background magnetic field, where the background magnetic field enters–in odd powers–to all orders. In particular, we calculate here $\Pi^5_{\mu\nu}$ in a magnetized medium and show that in the relevent momentum limit, this term contributes to neutrino effective charge. In order to bring this fact out, we would like to state that, $\Pi^5_{\mu\nu}$ in a magnetized medium can be written, in terms of general basis vectors available at hand and form factors, such as:

$$\Pi^5_{\mu\nu} = \epsilon_{\mu\nu 12}\left[k_\perp^2 f_1 + k_\parallel^2 f_2 + (k.u)f_3\right] + \epsilon_{\mu 12\alpha_\parallel}\left[k_{\nu_\parallel}u^{\alpha_\parallel}f_3\right.$$
$$+u_{\nu_\parallel}u^{\alpha_\parallel}f_4 + k^{\alpha_\parallel}u_{\nu_\parallel}f_5 + k_{\nu_\parallel}k^{\alpha_\parallel}f_6 + u_{\nu_\parallel}k^{\alpha_\parallel}f_7$$
$$\left.+k^{\alpha^\parallel}k_{\nu_\parallel}f_8\right] + g_{\mu\alpha_\parallel}k_{\nu_\perp}\left[u^{\tilde{\alpha_\parallel}}f_9 + k^{\tilde{\alpha_\parallel}}f_{10}\right] + g_{\mu\nu}k_{\alpha_\parallel}$$
$$\times k^{\tilde{\alpha}_\parallel}f_{11} + g_{\mu\nu}u_{\alpha_\parallel}k^{\tilde{\alpha}_\parallel}f_{12} + g_{\nu\alpha_\parallel}u^{\tilde{\alpha_\parallel}}k_{\mu_\perp}f_{13}$$
$$+g_{\nu\alpha_\parallel}k^{\tilde{\alpha_\parallel}}k_{\mu_\perp}f_{14} + g_{\nu\alpha_\parallel}k^{\tilde{\alpha_\parallel}}k_{\mu_\perp}f_{15}. \quad (35)$$

One can easily see that terms proportional to the product of u's is non zero in the static long wavelength limit, rendering the contribution finite.

V. EFFECTIVE NEUTRINO CHARGE.

It is evident from eq.(10) that to find the effective charge of the neutrino to odd orders in the external magnetic field we need only to calculate $\Pi^5_{\mu 0}$ in the limit $(k_0 = 0, \mathbf{k} \to 0)$.

From eqn.(23) it can be seen that in the above limit the integrand in Π^5_{00} contains terms which are proportional to p_0. As $k_0 = 0$ so the exponentials in the integrand in equation(22) are even in p_0 and so is $\eta_+(p_0)$, this makes the integrand in equation(22) odd in p_0 and thus Π^5_{00} vanishes.

Also

$$\mathrm{R}^{(o)}_{10} = -4i\eta_+(p_0)[p_3k_1(\tan(e\mathcal{B}s) - \tan(e\mathcal{B}s')) + k_3p_1\sec^2(e\mathcal{B}s)\tan(e\mathcal{B}s')] \quad (36)$$
$$\mathrm{R}^{(o)}_{20} = -4i\eta_+(p_0)[p_3k_2(\tan(e\mathcal{B}s) - \tan(e\mathcal{B}s')) + k_3p_2\sec^2(e\mathcal{B}s)\tan(e\mathcal{B}s')] \quad (37)$$

both of the above terms show that, after we integrate we will have terms proportional to k_3k_1 and k_3k_2 in Π^5_{10} and Π^5_{20} respectively, as the intrgrals in p_1 and p_2 are Gaussian, as we can see from the exponentials in equation(22). In the limit $\mathbf{k} \to 0$ this terms also go to zero.

In the specified limit of vanishing external photon momentum only Π^5_{30} remains to contribute for the charge.

$$\Pi^5_{30}(k_0 = 0, \vec{k} \to 0) = \lim_{k_0=0,\vec{k}\to 0} 4e^2\int\frac{d^4p}{(2\pi)^4}\int_{-\infty}^{\infty}ds\, e^{\Phi(p,s)}\int_0^\infty ds' e^{\Phi(p',s')}$$
$$\times\, \eta_+(p_0)\left[(k\cdot p)_\parallel - 2p_0^2\right](\tan(e\mathcal{B}s) + \tan(e\mathcal{B}s')). \quad (38)$$

Concentrating on equation(38), it is noted that except for the exponential factors the integrand is free of the perpendicular components of the momentum. This implies that the the perpendicular components of the loop momentum can be integrated out.The exponential factors can be written as,

$$\Phi(p,s) + \Phi(p',s') = \Phi_\parallel + \Phi_\perp\,, \quad (39)$$

where,

$$\Phi_\parallel = is\,(p_\parallel^2 - m^2) + is'\,(p_\parallel'^2 - m^2) - \varepsilon\,|s| - \varepsilon\,|s'|\,, \quad (40)$$
$$\Phi_\perp = -\frac{i\tan e\mathcal{B}s}{e\mathcal{B}}\,p_\perp^2 - \frac{i\tan e\mathcal{B}s'}{e\mathcal{B}}\,p_\perp'^2\,. \quad (41)$$

Therefore, integration of the perpendicular part of the momentum gives us,

$$\int\frac{d^2p_\perp}{(2\pi)^2}\,e^{\Phi_\perp}$$
$$= \exp\left(-\frac{i}{e\mathcal{B}}\frac{\tan e\mathcal{B}s\tan e\mathcal{B}s'}{\tan e\mathcal{B}s + \tan e\mathcal{B}s'}k_\perp^2\right)\int\frac{d^2p_\perp}{(2\pi)^2}$$
$$\times\,\exp\left(-i\frac{\tan e\mathcal{B}s\tan e\mathcal{B}s'}{e\mathcal{B}}(p_\perp + \frac{\tan e\mathcal{B}s'}{\tan e\mathcal{B}s + \tan e\mathcal{B}s'}k_\perp)^2\right)$$
$$\to -\frac{1}{4\pi}\frac{ie\mathcal{B}}{\tan e\mathcal{B}s + \tan e\mathcal{B}s'}\,; \quad (42)$$

where we have neglected terms up-to $\mathcal{O}(k_\perp^2)$ since, to calculate the effective charge of the neutrinos we ultimately have to take the limit $\mathbf{k} \to 0$. Hence, eq.(38) can be written as

$$\Pi^5_{30}(k_0 = 0, \vec{k} \to 0) = -\lim_{k_0=0,\vec{k}\to 0}\frac{ie^3\mathcal{B}}{\pi}\int\frac{d^2p_\parallel}{(2\pi)^2}\int_{-\infty}^{\infty}ds\,e^{is(p_\parallel^2 - m^2) - \varepsilon|s|}$$
$$\times\int_0^\infty ds' e^{is'(p_\parallel'^2 - m^2) - \varepsilon|s'|}\eta_+(p_0)\left[(k\cdot p)_\parallel - 2p_0^2\right]. \quad (43)$$

Using the following relations,

$$\int_{-\infty}^{\infty}ds\,e^{is\,(p_\parallel^2 - m^2) - \varepsilon\,|s|}\int_0^\infty ds' e^{is'\,(p_\parallel'^2 - m^2) - \varepsilon\,|s'|} = 2\pi i\frac{\delta(p_\parallel^2 - m^2)}{(p_\parallel'^2 - m^2) + i\varepsilon} \quad (44)$$

we can write equation(43) as

$$\begin{aligned}
&\Pi^5_{30}(k_0=0,\vec{k}\to 0)\\
&=\lim_{k_0=0,\vec{k}\to 0} 2e^3\mathcal{B}\int \frac{d^2p_\parallel}{(2\pi)^2}\frac{(p_\parallel^2-m^2)}{(p'^2_\parallel-m^2)+i\varepsilon}\\
&\times\,\eta_+(p_0)\left[(k\cdot p)_\parallel-2p_0^2\right]. \qquad (45)
\end{aligned}$$

The equation above consists of two integrals, which can be worked out to give,

$$\begin{aligned}
&\lim_{k_0=0,\vec{k}\to 0}\int \frac{d^2p_\parallel}{(2\pi)^2}(2\pi)\frac{(p_\parallel^2-m^2)}{(p'^2_\parallel-m^2)+i\varepsilon}\eta_+(p_0)(k\cdot p)_\parallel\\
&=\frac{1}{2}\int\frac{dp}{2\pi}\,\frac{\eta_+(E_p)}{E_p} \qquad (46)
\end{aligned}$$

$$\begin{aligned}
&\lim_{k_0=0,\vec{k}\to 0}\int \frac{d^2p_\parallel}{(2\pi)^2}(2\pi){p_0}^2\frac{(p_\parallel^2-m^2)}{(p'^2_\parallel-m^2)+i\varepsilon}\eta_+(p_0)\\
&=\frac{1}{4}\int\frac{dp}{2\pi}\,[\frac{\eta_+(E_p)}{E_p}-\beta\eta_+(E_p)] \qquad (47)
\end{aligned}$$

where $E_p^2=p^2+m^2$. Now combining the above results with that from equation(45) we get

$$\Pi^5_{30}(k_0=0,\vec{k}\to 0)=\frac{(e^3\mathcal{B})}{2\pi}\beta\int\frac{dp}{2\pi}\eta_+(E_p). \qquad (48)$$

After taking the trace in equation(10) we get

$$e^\nu_{\rm eff}=-\frac{G_F}{\sqrt{2}e}\,g_{\rm A}\,(1-\lambda)\,\Pi^5_{30}(k_0=0,\mathbf{k}\to 0)\cos\theta\,, \qquad (49)$$

where, θ is the angle between the magnetic field and the direction of the neutrino propagation. Finally when $m>\mu$ using Eq.(48) in Eq.(49) we obtain:

$$\begin{aligned}
e^\nu_{\rm eff}&=\frac{1}{8\sqrt{2}\pi^3}\,e^2\,G_F\,g_{\rm A}\,(1-\lambda)\,\mathcal{B}\,\cos\theta\\
&\times\sum_{n=0}^{\infty}(-1)^n\,(1+n)\,\beta m\,K_1[(n+1)\beta m]\\
&\times\,\cosh((n+1)\beta\mu)\,. \qquad (50)
\end{aligned}$$

To get a feeling of order of magnitude of the induced neutrino charge, we can take the ratio of the two with and without magnetica field. In the limit of vanishing chemical potential, it turns out to be:

$$\frac{e^\nu_{eff}(\mathcal{B})}{e^\nu_{eff}(\mathcal{B}=0)}=\frac{c_A}{4\pi^3}\Big(\frac{\mathcal{B}}{\mathcal{B}_c}\Big)(m\beta)^3K_1(m\beta)\cos\theta \qquad (51)$$

As one can see from eqn. [51] that, the ratio $\frac{\mathcal{B}}{\mathcal{B}_c}$ hold the key to the question whether the neutrino effectie charge can dominate that in a unmagnetized plasma. Since for low temperature (as is the case with) stars, all the factors except the ratio, can be taken to be of order unity, then this ratio would determine whether it dominate neutrino effective charge or not. We believe that since for magnetars this ratio is far larger than one, so there the neutrino effective charge may have some interesting role to play.

VI. CONCLUSION

To conclude, we note that only the left-handed neutrinos acquire an effective charge. The induced charge for the right-handed neutrinos vanish since they have no weak interactions which mediate the effective electromagnetic interaction.

Coming to more important issues, we note that, the presence of a magnetic field breaks the isotropy of the system and introduces a preferential direction as a consequence neutrinos propagating along the direction of the magnetic field acquire a positive charge whereas those propagating in the opposite direction acquire a negative charge. The net effect of this would then be the creation of a charge current along the direction of the field.

Interestingly, neutrinos propagating in a direction perpendicular to the field would acquire zero effective charge. This is in direct contrast with the case of a thermal medium in absence of an external magnetic field. In a thermal medium neutrinos acquire an effective charge irrespective of their direction of propagation. Therefore for isotropic neutrino propagation no net current is generated in that case unlike in the presence of a magnetic field.

It is worth mentioning here that a possible application of our result could be to find out the generation magnetic field of neutron stars being produced in supernova explosions as well as neutrino wind driven instability responsible for blowing up the outer mantle of supernova as put forward by in [23].

The basic mechanism behind this process is as follows: imagine the collision of two media having some ensemble of plasma where one of them moves with a different center of mass velocity with respect to the other. For such a system if one tries to find out the dispersion relation of the plasmons, then, apart from the usual dispersion relation for the same there appears a new dispersion relation that depends on the velocity with which the other one is colliding with the one at rest. This is a well known situation discussed in many text books of plasma physics, for instance one can refer to , [31,32]. Depending on the situation, this new dispersion relation can show damping or instability of the system and that would be a function of the difference in velocity between the two medium.

A full relativistic analysis of such system would require the formalism of Non-equilibrium quantum statistical field theory, where the velocity of one of the media can be taken to be of the form, $u^\mu=(1,\vec{0})$ (medium at rest) and for the other the velocity is V^μ where more than one component of the vector V^μ should be different from zero (the medium in motion). The growth rate of the plasmons, in such systems has been estimated using formalism of Finite Temperature quantum statistical field theory [24] [25] as well as [23]. It is important to note though, that the finite temperature statistical physics techniques [24] [25] show the damping/growth rate coming from this new dispersion relation to be proportional to G_F^2. On the other hand the growth rate for the unstable plasmon mode– calculated using plasma physics techniques, show the same scaling as G_F [23].We plan to take up this issue in a future publication.

In conclusion, we have calculated the effective charge of neutrinos in a weakly magnetized plasma, in the limit $m > \mu$. It is observed that the neutrino charge acquires a direction dependence as a result of the presence of an external magnetic field and it is also proportional to the magnitude of the field strength present in the system.

ACKNOWLEDGMENT

We would like to thank Prof. N. D. Hari Dass and Prof. Palash B. Pal for helpful discussions. We also thank Prof. Patrick Aurenche for for going through the preliminary draft of the same, and pointing out few important points. AKG would like to thank Laboratoire de Physique Theoretique, Annecy, France for supporting a visit where a part of this work was carried out.

[1] G. G. Raffelt, *Stars as Laboratories for Fundamental Physics*, (University of Chicago Press, 1996)

[2] Duncan Lorrimer, "Binary and Milisecond Pulsars" in Living Reviews in Relativity. [http://www.livingreviews.org/Articles/ index.html] and references therein.

[3] E. P. Mazets, et al. Nature, **282**,587 (1979).

[4] Murakami, et al. Nature, **368**,432 (1984).

[5] C. Kouveliotou, et al.Nature, **391**,235 (1999)

[6] R. T. Duncan, C. Thompson, Astrophys. J. **392**, L9 (1992), C. Thompson, R. T. Duncan, Astrophys. J. **408**, 194 (1993), M. Ruderman, J. de Physique suppl. **41**, c2-125 (1980)

[7] C. Kouveliotou, T. Strohmayer, K. Hurley, J. van Paradijs, M. H. Finger, S. Dieters, P. Woods, C. Thompson, R. T. Duncan, Astrophys. J. **581**, L103 (1999)

[8] G. Chanmugam, Ann. Rev. Astron. & Astrophys. **30**, 143 (1992)

[9] L. L. DeRaad Jr., K. A. Milton and N. D. Hari Dass, Phys. Rev. D **14**, 3326 (1976)

[10] J. C. D'Olivio, J. F. Nieves and P. B. Pal, Phys. Rev. D **40**, 3679 (1989)

[11] A. N. Ioannisian and G. G. Raffelt, Phys. Rev. D **55**, 7038 (1997), hep-ph/9612285

[12] S. J. Hardy and D. B. Melrose, Publ. Astron. Soc. Aus. **13**, 144 (1996)

[13] V. N. Oraevsky, V. B. Semikoz and Ya. A. Smorodinsky, JETP Lett. **43**, 709 (1986)

[14] R. Shaisultanov, Phys. Rev. D **62**, 113005 (2000), hep-th/0002079

[15] H. Gies and R. Shaisultanov, Phys. Rev. D **62**, 73003 (2000), hep-ph/0003144

[16] H. Gies and R. Shaisultanov, Phys. Lett. B **480**, 129 (2000), hep-ph/0009342

[17] Neutrino Photon Effective Lagrangian In A Magnetized medium, Sushan Konar, Avijit K. Ganguly, Kaushik Bhattacharya (Under Preparation).

[18] J. F. Nieves and P. B. Pal, Phys. Rev. D **49**, 1398 (1994)

[19] T. Altherr and P. Salati, Nuc. Phys. B **421**, 662 (1994)

[20] J. Cooperstein, Phys. Rep. **163**, 95 (1988); H. A. Bethe, Rev. Mod. Phys. **62**,801 (1990); S. Bludman, Da Hsuan Feng, Th. Gaisser and S. Pittel, Phys. Rep. **256**, 1 (1995)

[21] R. C. Duncan, S. L. Shapiro and I. Wasserman, Astrophys. J. **309**, 141 (1986); S. A. Colgate and R. H. White, Astrophys. J. **143**, 626 (1986); H. A. Bethe and J. R. Wilson ibid **295**, 14 (1985); M Ramp and H. T. Janka, astro-ph/0005438

[22] van Paradijs, J., .Taam, R. E., and van den Heuvel, E .P .J , Astronomy and Astrophysics, **299**, L41.

[23] L. O. Silva et. al, Phys. Rev. Lett. **83**, 2703 (1999) and references therein.
R. Bingham, J. M. Dawson, J. J. Su, and H. A. Bethe, Phys. Lett. A 193, 279 (1994).

[24] .Luis Bento, Phys. Rev. D 61, 013004 (2000);

[25] Jose F. Nieves, Phys.Rev.D61:113008,2000; For a similar conclusion, using Relativistic Quantum Kinetic theory see: H. T. Elze, T. Kodama and R. Opher, Phys. Rev. D **63**, 013008 (2000).

[26] A. K. Ganguly, S. Konar and P. B. Pal, Phys. Rev. D **60**, 105014 (1999), hep-ph/9905206

[27] J. Schwinger, Phys. Rev. **82**, 664 (1951)

[28] W. Y. Tsai, Phys. Rev. D **10**, 1342 and 2699 (1974)

[29] W. Dittrich, Phys. Rev. D **19**, 2385 (1979)

[30] P. Elmfors, D. Grasso and G. Raffelt, Nucl. Phys. B **479**, 3 (1996)

[31] E. M. Lifshitz and L. P. Pitaevskii, "Physical Kinetics", Course of Theoretical Physics Volume 10, (Pergamon Press, New York 1981), p. 133.

[32] S. Ichimaru, Statistical Plasma Physics (Volume I), (Addison-Wesley, Massachusetts, 1992), pp.177.